Program Information								
	[Lesson Title] ***Volume of a Prism and Cylinder***			**TEACHER NAME** **Julie Thumann**		**PROGRAM NAME** **Cincinnati City Schools**	
	[Unit Title] ***Volume of 3-D Shapes***			**NRS EFL(s)** **1 – 5**		**TIME FRAME** **3, 75-minute classes**	
Instruction	**OBR ABE/ASE Standards – Mathematics**						
	Numbers (N)		**Algebra (A)**		**Geometry (G)**		**Data (D)**	
	Numbers and Operation		Operations and Algebraic Thinking		Geometric Shapes and Figures	**G.1.1** **G.1.2** **G.3.2** **G.4.2** **G.4.4**	Measurement and Data	**D.2.1** **D.2.2**
	The Number System		Expressions and Equations	**A.3.8** **A.3.9** **A.3.16**	Congruence		Statistics and Probability	
	Ratios and Proportional Relationships		Functions		Similarity, Right Triangles. And Trigonometry		**Benchmarks identified in red are priority benchmarks. Please see the Curriculum Alignments available on the Teacher Resource Center for a complete list of priority benchmarks and related Ohio ABLE lesson plans.*	
	Number and Quantity				Geometric Measurement and Dimensions	**G.5.2**		
					Modeling with Geometry			

Mathematical Practices (MP)			
✓	Make sense of problems and persevere in solving them. (MP.1)	✓	Use appropriate tools strategically. (MP.5)
✓	Reason abstractly and quantitatively. (MP.2)	✓	Attend to precision. (MP.6)
✓	Construct viable arguments and critique the reasoning of others. (MP.3)	✓	Look for and make use of structure. (MP.7)
✓	Model with mathematics. (MP.4)	✓	Look for and express regularity in repeated reasoning. (MP.8)

LEARNER OUTCOME(S)	**ASSESSMENT TOOLS/METHODS**
• Find the volume of a rectangular prism and a cylinder (three-dimensional objects) • Find a missing dimension when given the volume of a rectangular prism or cylinder	• My Favorite No • Checks for understanding during video • Student responses during class work • Teacher observation • Summative assessment: *Volume of Prisms and Cylinders* Assessment

LEARNER PRIOR KNOWLEDGE

- Students should be able to solve for the area of a rectangle and a circle
- Students should have knowledge of the TI-30XS calculator
- Students should be familiar with the GED Mathematics Formula Sheet

INSTRUCTIONAL ACTIVITIES	**RESOURCES**
1. Warm-up Activity: “My Favorite No” a. Pass out index cards/scrap paper and ask students to complete the following problems (remind students to show all their work!): i. Find the area of the circle with a diameter of 4	Index cards/scrap paper for student use Projector, ability to project Chalkboard or whiteboard

	meters. ii. Find the area of the rectangle with the length of 7 centimeters and a width of 6 centimeters. iii. Collect cards and makes two piles – "Yes" and "No" (make a mental note of who has the correct answers as you collect the cards because you can pair them with students for additional help). Review the "No" pile and chose one for each problem to review. Look for cards with similar mistakes. iv. Write the mistake on the board and discuss the positives of the problem and then how to correct the mistake. 2. Explain volume. a. Volume, also called capacity, is the measure of space inside a three-dimensional object. You measure volume in cubic units. In other words, if the sides of an object are measured in inches, the volume is the number of cubic inches you would need to fill the object. 3. Watch Find the volume of a rectangular prism by developing a formula video (5:12) as a class. a. Before the video is played, ask students to have their notebooks and pencils ready to take notes, but only when you pause the video. Otherwise, they should be watching and listening to the video. i. Write the lesson title on the board and ask the students to write this on a new page in their notes: Developing the formula for volume of a rectangular prism. b. Introduction Video Discussion: knowing how to make effective use of space is a useful skill in everyday life. In some careers, it's a skill that can save you money. For example, after goods are made in a factory, they need to be shipped or transported. Many times these goods are moved by trucks. In the video we are about to watch, we will develop the	Computer with Internet access Find the volume of a rectangular prism by developing a formula. (n.d.). Retrieved from https://learnzillion.com/assignments/7P2PS9U Student copies of *Volume Foldable* handout (attached) Student copies of Math Talk Bookmark (attached) Math Talk Bookmark. (n.d.). Retrieved from https://www.pinterest.com/pin/30751209929886153/ Student copies of *Mathematics Formula Sheet & Explanation* (attached) Mathematics Formula Sheet & Explanation [PDF file]. (n.d.). Retrieved from http://www.gedtestingservice.com/uploads/files/0756c16704434ff71e43c8117a5fa738.pdf TI-30XS calculators for student use Student copies of *Popcorn Cylinders Anyone?* (attached) *Popcorn Cylinders Anyone?* answer key (attached) Popcorn Cylinders Anyone? [PDF file]. (n.d.). Retrieved from https://illuminations.nctm.org/uploadedFiles/Content/Lessons/Resources/6-8/Popcorn-AS-Cylinders.pdf 8.5x11" white and colored paper for student use

		formula for volume of a rectangular prism using the scenario of a moving truck and boxes. c. Teacher should watch the video prior to the lesson so they may determine appropriate places to pause the lesson. Please refer to the following times as pausing guidelines; however, it will vary from class or individual instruction. i. Pause 0:25 – Inform students to be thinking and their prior knowledge – be thinking about how the warm-up relates and how this will be an extension to that knowledge. ii. Pause 0:57 – How can we compare and contrast area and volume? Teacher records their answers on the board. Ask students to respond with "thumbs up/thumbs down" for understanding. iii. Pause 1:20 – Instruct students to have their notebooks and pencils ready for notes. iv. Pause 1:32 – Instruct students to write down this question. v. Pause 2:12 – Students should draw the rectangular prism in their notes. vi. Pause 2:30 – Now, add the length, width, and height to their rectangular prism. vii. Pause 3:40 – What does this bottom layer represent? Area or perimeter? Ask students to prove their answer or provide evidence. Ask students to respond with "thumbs up/thumbs down" for comprehension. viii. Pause 4:13 – Ask students, "Could we conclude that volume of a rectangular prism is area of the base and then stacked?" Why or why not? Ask students to respond with "thumbs up/thumbs down" for comprehension. ix. Pause 5:02 – Ask students to write the volume formula: V = lwh. Do you see another formula	Tape for student use Popcorn for student use Paper plates for student use Cups for student use Rulers for student use Extend understanding of volume of prisms to volume of cylinders (C). (n.d.). Retrieved from https://learnzillion.com/assignments/VP55NF7 Student copies of A, B, C Assessment (attached) Student copies of *Volume of Prisms and Cylinders* Assessment (attached) Zike, D. (n.d.). *Teaching Mathematics with Foldables*. Retrieved from https://blogs.edutech.nodak.edu/badlandsreadingcouncil/files/2012/03/math-foldables.pdf

	inside of this formula? Is this statement true: area of a rectangle times the height equals volume of a rectangular prism? x. Complete video and discuss any additional questions from the class. 4. Pass out the *Volume Foldable* handout a. Model folding instructions for a shutter foldable and provide supplies for students to create the foldable. a. Demonstrate how to solve the Model Problems using a think-aloud strategy b. Once you feel students understand how to solve the Model Problems, ask students to participate in the problem-solving process (use phrases from the *Math Talk Bookmark* to solicit student responses and check student understanding). i. Provide students the *Mathematics Formula Sheet & Explanation* and a TI-30XS calculator. ii. Project the template on the board and solve the modeling problems together. iii. Provide students time to complete the practice questions independently or with a partner. 5. *Popcorn Cylinders Anyone?* a. For this activity, students will be comparing the volume of 2 cylinders created using the same sheet of paper. Students will be determining which can hold more popcorn. It's all about using the volume to determine the size of the container. a. Distribute *Popcorn Cylinders Anyone?* and materials to students. a. Have students work together to complete the activity. Walk around the room to address student questions and comment on student work. b. At the end of the activity, review the questions as a class. i. Starting with question #4, choose different pairs	

<table>
<tr><td></td><td>or groups to share their answers or solutions, and ask the rest of the class if they agree or disagree.
ii. Discuss the correct answer and record them on the board.
6. Watch Extend understanding of volume of prisms to volume of cylinders video (approximately 6 minutes)
a. This video contains 10 questions to be reviewed and discussed with students.
b. Use A, B, C Assessment to review the 10 questions.
7. Have students complete the Volume of Prisms and Cylinders Assessment
a. Students complete the assessment and then review as a class or turn in to teacher for evaluation.</td><td></td></tr>
<tr><td></td><td colspan="2">DIFFERENTIATION

• Provide students with partially complete handout, graphic organizer, and/or foldables
• Display written vocabulary terms and definitions
• Allow students to work individually, in pairs, or in class groups
• During video: provide script, guided notes</td></tr>
<tr><td rowspan="2">Reflection</td><td colspan="2">TEACHER REFLECTION/LESSON EVALUATION</td></tr>
<tr><td colspan="2">ADDITIONAL INFORMATION</td></tr>
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Volume of A
Cylinder
Example #1

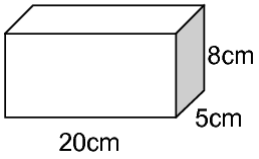
Volume of A
Cylinder
Example #2

Volume of A
Rectangular Prism
Example #1

Volume of A
Rectangular Prism
Example #2

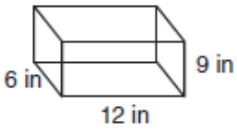
Model Problems

Determine the volume of the rectangluar prism.



Practice Problems

Determine the volume of the rectangular prism.



Model Problem

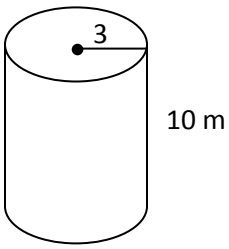
The volume of a rectangular solid is 210 cubic centimeters, the height is 15 centimeters, and the width is 2 centimeters. Find the length.

Practice Problem

A box in the shape of a cube has a volume of 64 cubic inches. What is the length of a side of the box?

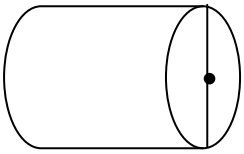
Model Problem

Determine the volume of the cylinder to the nearest tenth.



Practice Problem

Determine the volume of the cylinder to the nearest hundreth.



Model Problem

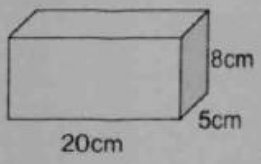
Find the diameter, to the nearest integer, of a cylinder with a height of 8.5 cm and a volume of 667.59 cm^3.

Practice Problem

Find the height, to the nearest tenth, of a cylinder with a radius of 8 cm and a volume of 148.12 cm^3.

Model Problems

Determine the volume of the rectangluar prism.



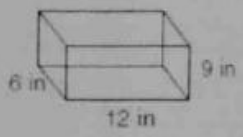
$V = l \cdot w \cdot h$

$V = 8 \cdot 5 \cdot 20$

$V = 800\ cm^3$

Practice Problems

Determine the volume of the rectangular prism.



$V = l \cdot w \cdot h$

$V = 6 \cdot 9 \cdot 12$

$V = 648\ in^3$

Model Problem

The volume of a rectangular solid is 210 cubic centimeters, the height is 15 centimeters, and the width is 2 centimeters. Find the length.

$V = l \cdot w \cdot h$

$210 = l(2)(15)$

$210 = 30l$

$7 = l$

Practice Problem

A box in the shape of a cube has a volume of 64 cubic inches. What is the length of a side of the box?

$V = l \cdot w \cdot h$

$64 = s^3$

$s = 4$

$s = 1 \quad V = 1$

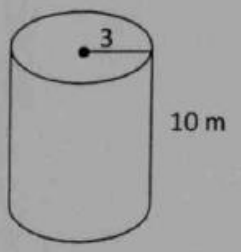
$s = 2 \quad V = 8$

$s = 3 \quad V = 27$

$s = 4 \quad V = 64$

Model Problem

Determine the volume of the cylinder to the nearest tenth.



$V = \pi r^2 h$

$V = \pi(3)^2(10)$

$V = 282.7\ m^3$

Model Problem

Find the diameter, to the nearest integer, of a cylinder with a height of 8.5 cm and a volume of 667.59 cm^3.

$V = \pi r^2 h$

$667.59 = \pi r^2(8.5)$

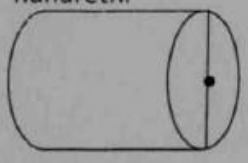
$25 = r^2$

$5 = r$

$d = 10$

Practice Problem

Determine the volume of the cylinder to the nearest hundreth.



12 in

$V = \pi r^2 h$

$V = \pi(1.25)^2(12)$

$V = 58.90\ in^3$

Practice Problem

Find the height, to the nearest tenth, of a cylinder with a radius of 8 cm and a volume of 148.12 cm^3.

$V = \pi r^2 h$

$148.12 = \pi(8)^2 h$

$.7 = h$

Math Talk

- I agree/disagree with you because...
- What I heard you say was...
- What key words helped you solve this?
- Can you explain this to me?
- What were you thinking here?
- How did you solve it?
- What did you start with?
- Why did you choose that operation?
- What strategy did you use?
- Why did you choose that strategy?
- How did you know your answer was right?
- Prove your answer is right.
- How else can you solve it?
- How did this help you understand?
- How is this like other problems you've solved?

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Mathematics Formula Sheet & Explanation

The 2014 GED® Mathematical Reasoning test contains a formula sheet, which displays formulas relating to geometric measurement and certain algebra concepts. Formulas are provided to test-takers so that they may focus on *application*, rather than the *memorization*, of formulas.

Area of a:

square	$A = s^2$
rectangle	$A = lw$
parallelogram	$A = bh$
triangle	$A = \frac{1}{2}bh$
trapezoid	$A = \frac{1}{2}h(b_1 + b_2)$
circle	$A = \pi r^2$

Perimeter of a:

square	$P = 4s$
rectangle	$P = 2l + 2w$
triangle	$P = s_1 + s_2 + s_3$
Circumference of a circle	$C = 2\pi r$ OR $C = \pi d$; $\pi \approx 3.14$

Surface area and volume of a:

rectangular prism	$SA = 2lw + 2lh + 2wh$	$V = lwh$
right prism	$SA = ph + 2B$	$V = Bh$
cylinder	$SA = 2\pi rh + 2\pi r^2$	$V = \pi r^2 h$
pyramid	$SA = \frac{1}{2}ps + B$	$V = \frac{1}{3}Bh$
cone	$SA = \pi rs + \pi r^2$	$V = \frac{1}{3}\pi r^2 h$
sphere	$SA = 4\pi r^2$	$V = \frac{4}{3}\pi r^3$

(p = perimeter of base with area B; $\pi \approx 3.14$)

Data

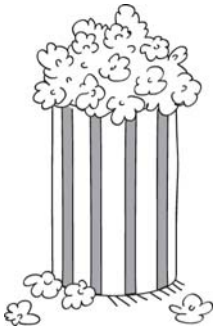
mean	mean is equal to the total of the values of a data set, divided by the number of elements in the data set
median	median is the middle value in an odd number of ordered values of a data set, or the mean of the two middle values in an even number of ordered values in a data set

Algebra

slope of a line	$m = \frac{y_2 - y_1}{x_2 - x_1}$
slope-intercept form of the equation of a line	$y = mx + b$
point-slope form of the equation of a line	$y - y_1 = m(x - x_1)$
standard form of a quadratic equation	$y = ax^2 + bx + c$
quadratic formula	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
Pythagorean theorem	$a^2 + b^2 = c^2$
simple interest	$I = Prt$ (I = interest, P = principal, r = rate, t = time)
distance formula	$d = rt$
total cost	total cost = (number of units) × (price per unit)

Popcorn Cylinders Anyone?

NAME _____

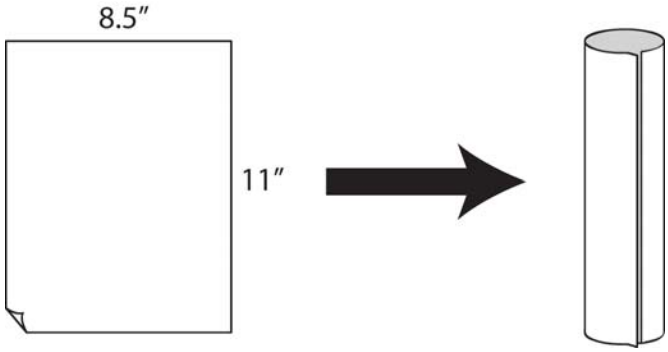


For this activity you will be comparing the volume of 2 cylinders created using the same sheet of paper. You will be determining which can hold more popcorn. To do this, you will have to find a pattern for the dimensions for containers.

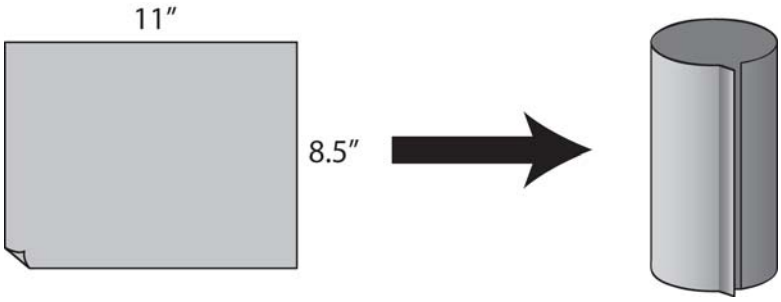
Materials:

- 8.5×11 in. white paper
- 8.5×11 in. colored paper
- Tape
- Popcorn
- Plate
- Cup
- Ruler

Take the white paper and roll it up along the longest side to form a baseless cylinder that is tall and narrow. Do not overlap the sides. Tape along the edges. Measure the dimensions with a ruler and record your data below and on the cylinder. Label it Cylinder A.



Take the colored paper and roll it up along the shorter side to form a baseless cylinder that is short and stout. Do not overlap the sides. Tape along the edge. Measure the height and diameter with a ruler and record your data below and on the cylinder. Label it Cylinder B.



1.

DIMENSION	CYLINDER A	CYLINDER B
HEIGHT (in.)		
DIAMETER (in.)		
RADIUS (in.)		

2. Do you think the two cylinders will hold the same amount? Do you think one will hold more than the other? Which one? Why?

3. Place Cylinder B on the paper plate with Cylinder A inside it. Use your cup to pour popcorn into Cylinder A until it is full. Carefully, lift Cylinder A so that the popcorn falls into Cylinder B. Describe what happened. Is Cylinder B full, not full, or overflowing?

As you share your popcorn snack, answer the questions below.

4. a) Was your prediction correct? How do you know?

b) If your prediction was incorrect, describe what actually happened.

5. a) State the formula for finding the volume of a cylinder.

b) Calculate the volume of Cylinder A? Label the dimensions in the figure.

c) Calculate the volume of Cylinder B? Label the dimensions in the figure.

d) Explain why the cylinders do or do not hold the same amount. Use the formula for the volume of a cylinder to guide your explanation.

6. Which measurement impacts the volume more: the radius or the height? Work through the example below to help you answer the question.

a) Assume that you have a cylinder with a radius of 3 inches and a height of 10 inches. Increase the radius by 1 inch and determine the new volume. Then using the original radius, increase the height by 1 inch and determine the new volume.

CYLINDER	RADIUS	HEIGHT	VOLUME
ORIGINAL	3	10	
INCREASED RADIUS			
INCREASED HEIGHT			

b) Which increased dimension had a larger imact on the volume of the cylinder? Why do you think this is true?

7. By how much would you have to decrease the height of Cylinder B to make the volumes of the two prisms equal?

8. Compare and contrast your results from the prism activity and the cylinder activity. What conclusions can you make about the relationship between dimensions, area, and volume?

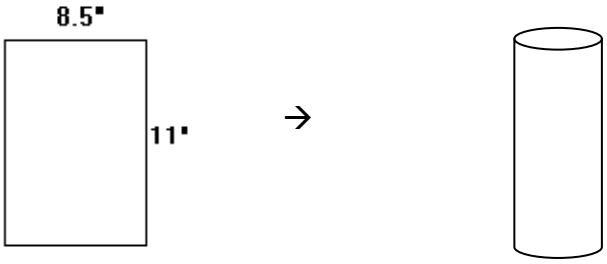
Answer Key – Popcorn Cylinders Anyone?

For this activity you will be comparing the volume of 2 cylinders created using the same sheet of paper. You will be determining which can hold more popcorn. To do this, you will have to find a pattern for the dimensions for containers.

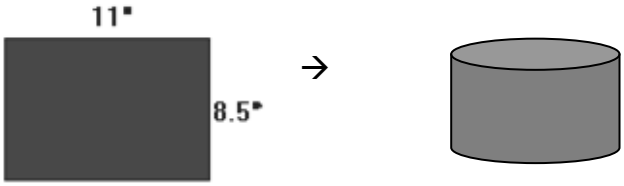
Materials:

- 8.5 inch by 11 inch white paper
- 8.5 inch by 11 inch colored paper
- Tape
- Popcorn
- Plate
- Cup
- Ruler

Take the white paper and roll it up along the longest side to form a baseless cylinder that is tall and narrow. Do not overlap the sides. Tape along the edge. Measure the dimensions with a ruler. Record your data below and on the cylinder. Label it Cylinder A.



Take the colored paper and roll it up along the shorter side to form a baseless cylinder that is short and stout. Do not overlap the sides. Tape along the edge. Measure the height and diameter with a ruler. Record your data below and on the cylinder. Label it Cylinder B.



1.

DIMENSION	CYLINDER A	CYLINDER B
HEIGHT (in.)	[11 in]	[8.5 in]
DIAMETER (in.)	[~2.7 in]	[~3.5 in]
RADIUS (in.)	[~1.4 in]	[~1.8 in]

2. Do you think the two cylinders will hold the same amount? Do you think one will hold more than the other? Which one? Why?

Answers will vary.

3. Place Cylinder B on the paper plate with Cylinder A inside it. Use your cup to pour popcorn into Cylinder A until it is full. Carefully, lift Cylinder A so that the popcorn falls into Cylinder B. Describe what happened. Is Cylinder B full, not full, or overflowing?

Cylinder B is not full. There is still room in the cylinder for more popcorn.

As you share your popcorn snack, answer the questions below.

4. a) Was your prediction correct? How do you know?

Answers will vary.

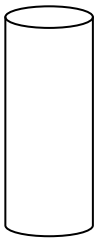
b) If your prediction was incorrect, describe what actually happened.

Cylinder B has a greater volume than Cylinder A.

5. a) State the formula for finding the volume of a cylinder.

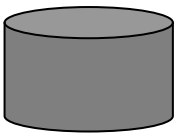
$V = \pi r^2 h$

b) Calculate the volume of Cylinder A? Label the dimensions in the figure.



$V = \pi r^2 h \approx \pi(1.4)^2(11) \approx 67.7 \text{ in}^3$

c) Calculate the volume of Cylinder B? Label the dimensions in the figure.



$V = \pi r^2 h \approx \pi(1.8)^2(8.5) \approx 86.5 \text{ in}^3$

d) Explain why the cylinders do or do not hold the same amount. Use the formula for the volume of a cylinder to guide your explanation.

The cylinders have different radii and heights, so the volumes are different.

6. Which measurement impacts the volume more: the radius or the height? Work through the example below to help you answer the question.

a) Assume that you have a cylinder with a radius of 3 inches and a height of 10 inches. Increase the radius by 1 inch and determine the new volume. Then using the original radius, increase the height by 1 inch and determine the new volume.

CYLINDER	RADIUS	HEIGHT	VOLUME
ORIGINAL	3 in	10 in	[~282.7 in^3]
INCREASED RADIUS	[4 in]	[10 in]	[~502.7 in^3]
INCREASED HEIGHT	[3 in]	[11 in]	[~311.0 in^3]

b) Which increased dimension had a larger imact on the volume of the cylinder? Why do you think this is true?

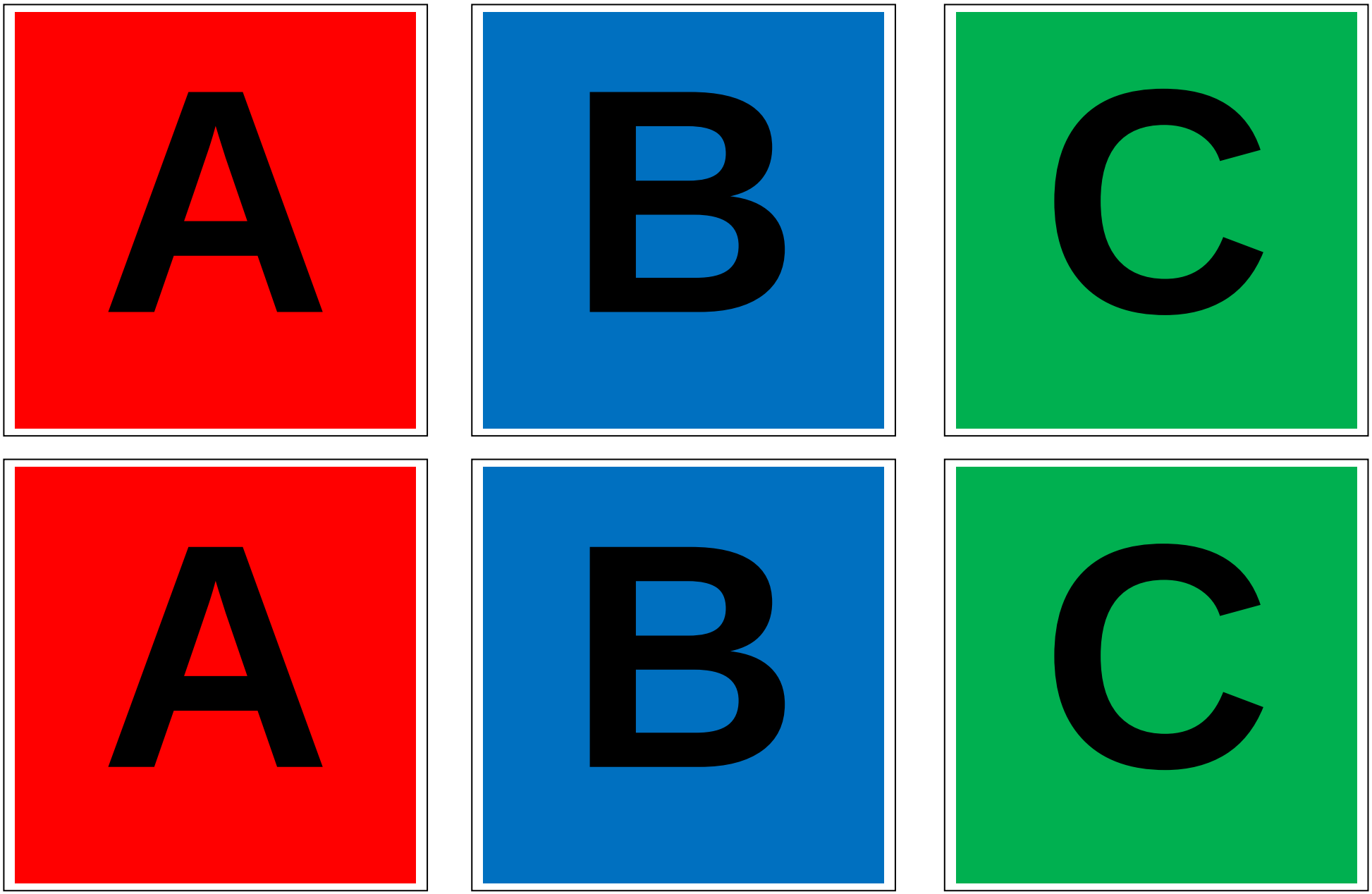
Increasing the radius increased the volume more than increasing the height. This is because the radius is squared to find the volume, which increases its impact on the volume.

7. By how much would you have to decrease the height of Cylinder B to make the volumes of the two prisms equal?

$$V_A \approx 67.7 \text{ in}^3$$
$$V_B \approx 67.7 \text{ in}^3 = \pi(1.8)^2(h)$$
$$h \approx 6.7 \text{ in}$$
The height would need to be decreased by about $8.5 - 6.7 \approx 1.8$ in.

8. Compare and contrast your results from the prism activity and the cylinder activity. What conclusions can you make about the relationship between dimensions, area, and volume?

Answers will vary. Students may point out the similarity in the volume formulas $V = l^2h$ and $V = \pi r^2 h$ and how this effected their results.


A
B
C
A
B
C

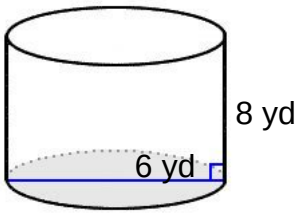
Name : _____ Score : _____

Teacher : _____ Date : _____

Volume of Prisms and Cylinders

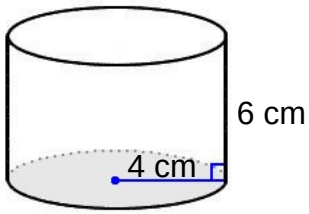
Find the volume for each figure. Round your answers to the nearest hundredth, if necessary.

1)



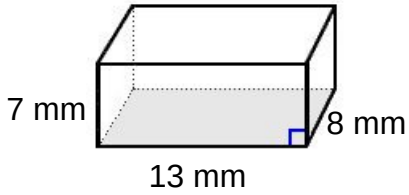
Volume: _____

2)



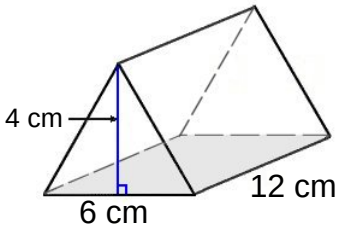
Volume: _____

3)



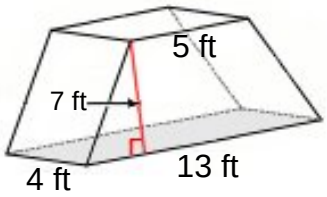
Volume: _____

4)



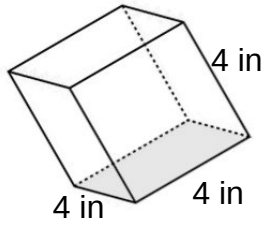
Volume: _____

5)



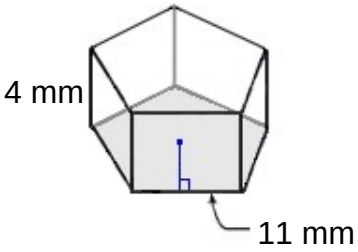
Volume: _____

6)



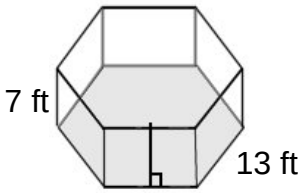
Volume: _____

7)



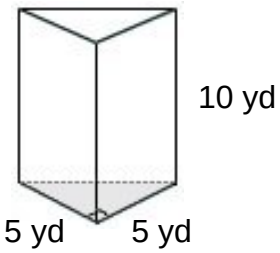
Volume: _____

8)



Volume: _____

9)



Volume: _____

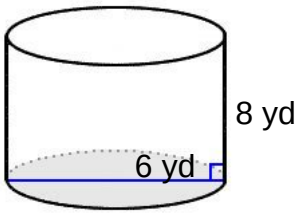
Name : _____ Score : _____

Teacher : _____ Date : _____

Volume of Prisms and Cylinders

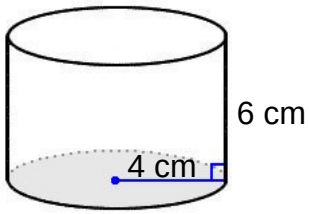
Find the volume for each figure. Round your answers to the nearest hundredth, if necessary.

1)



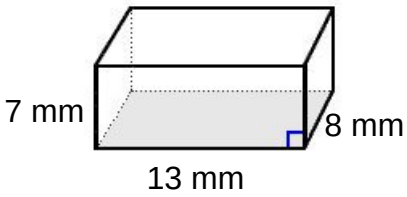
Volume: 226.19 yd^3

2)



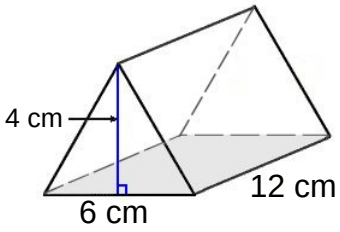
Volume: 301.59 cm^3

3)



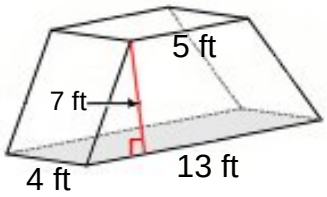
Volume: 728.00 mm^3

4)



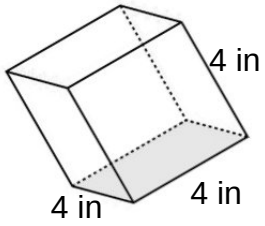
Volume: 144.00 cm^3

5)



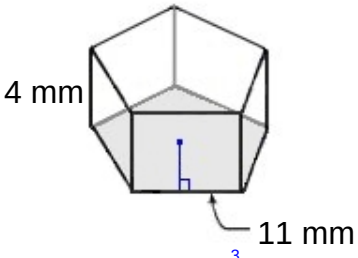
Volume: 252.00 ft^3

6)



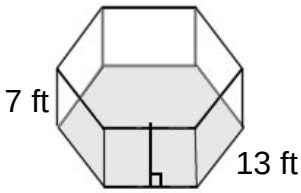
Volume: 64.00 in^3

7)



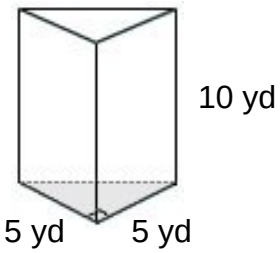
Volume: 832.71 mm^3

8)



Volume: 3073.52 ft^3

9)



Volume: 125.00 yd^3